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Testing Relativity and Gravitational Theories by Radar Ranging to a Heliocentric Satellite [and Discussion]

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Testing relativity and gravitational theories by radar ranging to a heliocentric satellite

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Laser ranging to the Moon and radar ranging to the planets and space probes are providing increasingly more accurate estimates of post-Newtonian gravitational effects. This paper summarizes the results obtained so far and outlines future possibilities of more accurate tests of relativity by laser and X and K band ranging to space probes, particularly to a highly eccentric or direct impact solar probe.

What is the correct theory of gravity? Newton's inverse square law is still adequate to describe most gravitational phenomena – from the orbit of the Moon to models of the Universe – but it has been replaced by Einstein's general relativity which not only reduces to Newton's theory for weak fields and slow motion, but successfully explains the minute departures from Newton's theory in the orbits of the planets. Is Einstein's theory correct? There is no shortage of alternative theories – the scalar tensor theory of Brans & Dicke, vector–tensor theories, Birkhoff's or Whitehead's special relativistic theories . . ., how can we decide between them?

Since gravity is a weak force, we need strong gravitational fields – high velocities, large distances. These are not readily available to the experimentalist; he is forced to remain within the Solar System, using as accurate techniques as he can in weak field conditions. Radar ranging to celestial bodies – natural or artificial, offers one of the very few tools at his disposal; with them he can in principle differentiate between different theories, and perhaps find new effects that are not predicted by any theory. In this article I outline just a few such possibilities: the best hope I believe is to send an artificial satellite very close to the Sun to the region of highest gravitational field in the solar system.

1. EINSTEIN'S THEORY

General relativity is based on the conception that there exist a set of field variables, the metric tensor $g_{ij}(x)$ varying with position such that the motion of test bodies move on geodesics of the space with metric

$$\mathrm{d}s^2 = g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j, \tag{1}$$

where atomic clocks measure the proper time ds and light moves on null geodesics, ds = 0. This is usually referred to as a curve space–time theory, but it should be clear that no experiment can measure the curvature of space, only the behaviour of massive bodies and photons in space. The geometrical language used in relativity is just a mathematically convenient way of expressing the theory; what matters for the empiricist is the prediction for the outcome of a particular experimental procedure – does the prediction agree with experience?

The theory has to be completed by the introduction of 'field equations', that is, a rule for determining the field g_{ij} in terms of the sources, in this case massive bodies – or energy and momentum. These take the form

$$R_{ij} = \frac{8\pi G}{c^4}\,(T_{ij} - \tfrac{1}{2}g_{ij}\,T), \tag{2}$$

where R_{ij} (the Ricci curvature tensor) is linear in the second derivatives of the field g_{ij} and T_{ij} is the energy momentum tensor. The equation of motion of a test particle is then governed by equations

$$\frac{d^2x^i}{ds^2} = -\Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}, \tag{3}$$

where Γ^i_{jk} are linear in the first derivatives of the field g_{ij}. Equations (2) and (3) are just the Einstein analogues of Poisson's equation and the equation of motion in Newtonian theory, where the field is the scalar potential ϕ.

2. WHAT TO TEST

There are several features of the currently favoured theory that need to be confronted with experimental data:

(i) Are the field equations (2) correct, or should they be replaced by some other analogue of Poisson's equation?

(ii) Do atomic clocks actually keep the proper time ds? Do different clocks (basic transition, fine structure, hyperfine) keep different time?

(iii) Is gravity correctly described by a metric theory, i.e. are (1) and (3) correct for some g_{ij}, or does gravity need a different mathematical framework for its description?

(iv) Is there some interaction between local physics and the large scale structure of the Universe. For example, does motion of the Solar System relative to the rest of the Universe (aether drift) have any empirical consequences, does the strength of gravity change as the Universe expands?

3. HOW TO TEST

The strongest gravitational field that is available to the experimental scientist is at the surface of the Sun where

$$\epsilon = \frac{2GM}{R_{\odot}c^2} = 4 \times 10^{-6}.$$

This small value is indicative of the weakness of gravitational forces and of the difficulty of achieving very sensitive experimental constraints. At the surface of the Earth, ϵ is three orders of magnitude smaller. The theory of gravity predicts the orbits of test bodies, the path of light rays and the behaviour of clocks in the presence of gravitating bodies. The maximum effect is, therefore, to be achieved by light rays grazing the solar surface, sending accurate clocks very close to the Sun and measuring the orbit of a satellite that goes as close to the Sun as possible. Secular effects like a change in the Newtonian constant of gravity G require measurements over a long period of time, directionally dependent measurements require the orbits of particles or photons to have different orientations relative to the Universe at different times – for example, to measure the 'bending of light' when the Earth is at different positions in its orbit.

In terms of orbit determination by radar ranging, the obvious solution is to send a satellite down to a perihelion distance of (say) $4R_{\odot}$ and from range and range-rate measurements to determine the orbit. Such a mission is currently in a mission definition stage with the European Space Agency – namely 'The solar probe'.

4. THE SOLAR PROBE

The concept of this mission is to send a satellite first to Jupiter from which one can obtain a sufficiently strong gravitational deflexion to redirect the probe towards the Sun. A direct launch towards the Sun is not possible – there are no launchers available that can counter the Earth's orbit velocity and let the probe drop into the Sun, but there are, of course, launchers capable of reaching Jupiter, from where it is easy to redirect the orbit to reach down to $4R_\odot$ at perihelion. In an ideal world, all that is then needed is to measure range and range rate to determine the orbit, but this is not an ideal world, and many problems have to be overcome.

(a) *Thermal problems*

Clearly an approach to $4R_\odot$ will heat the satellite to 2500 K unless it is adequately shielded. This can, in fact (or rather in theory!), be accomplished by a two- or three-layered graphite shield placed about $1\frac{1}{2}$ m in front of the satellite which sits in the shadow cone of the shield; the temperature can be kept down to 50 °C in this way.

(b) *Non-gravitational forces*

What is needed for testing gravitational theories is an accurate determination of a gravitational orbit – the real orbit will be determined by the combined action of gravitational forces and non-gravitational forces of radiation pressure, solar wind pressure, etc. The non-gravitational forces have either to be measured with an accelerometer such as the French Cactus accelerometer which has recently been successfully flown, or the motion of the satellite corrected as in the American Triad drag free satellite. The drag-free concept is particularly appealing and was studied in depth for an earlier European Space Agency proposal Sorel (now unfortunately dead!). The main satellite acts as a protecting shield for a test mass in the centre of the satellite; as the satellite moves under the influence of non-gravitational forces and gravitational forces, whereas the test mass only responds to the gravitational field alone, the relative position of the ball and satellite change. This change is measured, say by optical sensors, and the main satellite then shunted by firing gas jets so the ball is again in the centre. The whole satellite then follows a purely gravitational trajectory with only very minor random fluctuations. An accuracy of up to $10^{-12}\,\mathrm{m/s^2}$ is, in principle, achievable by this technique.

(c) *Communication*

Conventional radar ranging is in S band (2GHz) with the increasing availability of X band (8GHz). The S band in particular is seriously affected by the solar wind plasma and it is unlikely that useful measurements closer than $20R_\odot$ can be achieved by a single S band communication system. The use of two frequencies radically improves the situation and probably permits ranging to $8R_\odot$ – I say probably because until the Mariner Jupiter Saturn Mission has flown, we have no hard empirical data by which to judge. Using standard ranging techniques, therefore, probably means a blackout period during which communication and range and range rate measurements are lost, it would therefore be necessary to store measurements during this period and send them back when communication is re-established. Alternatively, the orbit could have a perihelion distance of $8R_\odot$ so that communication can always be maintained.

An alternative is to use an even higher frequency, say K band or lasers. While round trip laser communication over 1 AU (*ca.* 150×10^9 m) may not be possible, one way, Earth to satellite, should be possible. Range measurements can then be achieved down to perihelion by having an on board clock and measuring the time of emission and time of arrival by a laser pulse, the information being stored during radar blackout and relayed back after blackout.

5. PURPOSE OF THE SOLAR PROBE

The functions of such a probe would not be solely or even primarily to test gravitational theories. A probe reaching down to $4R_\odot$ can make *in situ* measurements of the solar wind plasma, particles and fields, and provide exceedingly valuable data on the structure of the solar wind in the sub-Alvenic, and subsonic regimes, on the particle acceleration mechanisms, the plasma instabilities, electron and ionic temperatures, quantities that can only be inferred from theoretical models and measurements outside $50R_\odot$. This alone would, in my opinion, justify such a mission.

Secondly, the orbit analysis would determine the Newtonian solar quadropole moment J_2, a quantity which can otherwise only be inferred from solar oblateness results with all their problems of separating equatorial brightening from dynamical oblateness. A knowledge of the quadropole moment would be a valuable constraint on models of the solar interior. Is the Sun rotating uniformly or is the central region spinning faster than the surface, as I, among other theoreticians, currently argue? Our knowledge of the solar interior is so slight – and what confidence we had in our theoretical speculations so undermined by recent measurements of solar neutrinos and oscillations as well as lithium deficiency, that a measurement of the quadropole moment by itself would, in my opinion, justify such a mission. The whole of stellar structure and evolution theory and its wider application has been brought into question and we desperately need empirical data on the internal structure of the Sun.

Finally, such a mission could achieve more accurate tests of gravitational theory than any other way. By reaching into $4R_\odot$ where the field strength $\epsilon \approx 10^{-6}$, we could hope to test general relativity to an accuracy of between one part in 10^3 and one in 10^5 depending on the ranging and acceleration techniques used. Moreover, by equipping the satellite with accurate clocks, and by the time such a mission could fly in, say, 1982, these will probably be accurate to 1 part in 10^{14}, the theory could be tested to order ϵ^2, would we at this level uncover a change as radical as that from Newtonian to Einsteinian theory?

6. THE GRAVITATIONAL PARAMETERS

The now classical method of analysing gravitational theories is to accept the metric framework of general relativity but to drop the field equations, the metric around a spherical mass like the Sun is then expanded in the form

$$\mathrm{d}s^2 = \left[1 - \frac{2GM}{rc^2} - 2\beta\left(\frac{GM}{rc^2}\right)^2\right]\mathrm{d}t^2 - \frac{1}{c^2}\left(1 - 2\gamma\frac{GM}{rc^2} - 2\delta\left(\frac{GM}{rc^2}\right)^2\right)\mathrm{d}r^2$$

and the parameters β and γ determined from fitting the measurements of range and range rate to the parameterized orbit. Of course, such an orbit is also influenced by the quadropole moment J_2, and present experimental values are

$$J_2 < 10^{-6}, \quad \gamma = 1 \pm 0.02, \quad \beta = 1 \pm 0.05.$$

With the accuracy that can be achieved by a solar probe, these values can be improved by one to three orders of magnitude. Even at the minimum level, it is necessary to analyse the orbit to include terms of order $(v/c)^3$, where v is the velocity of the satellite; at this level one could test for aether drift by including terms like Vv^2/c^3, where V is the aether drift velocity, and look for non-metric contributions (if they exist) to the orbit. It is worth remarking here that such effects will only be found if they are included in the error analysis, since the error codes are essentially least squares fits to the included parameters.

7. Conclusion

The techniques and capabilities for more accurate tests of gravitation theories are now available. A solar probe going down to $4R_\odot$ perihelion with either on board acceleration measurements, or drag-free compensation and two frequency or laser ranging, could provide a significant jump in accuracy of testing Einstein's theory and in looking for effects that are not predicted by the theory – so testing the foundations of the theory as well as the field equations. A combined mission to undertake this aim and to measure solar wind particles and fields, and the solar quadropole moment, would be a great achievement.

Discussion

J. A. Weightman (*Geodetic Office*, *Elmwood Avenue*, *Feltham*, *Middlesex*). Since there will be communication difficulties at the terrestrial tracking stations when the solar probe satellite is only a few solar radii from the Sun, would there be merit in using Dr Vonbun's satellite-to-satellite tracking technique?

I. W. Roxburgh. The problem of ranging close to the Sun is the high electron density and its fluctuations in the inner solar wind, not the Earth's environment so I cannot at the moment see any advantage in using satellite-to-satellite tracking.